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QUANTIFICATION OF AGILITY OF A SUPPLY CHAIN USING FUZZY LOGIC

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Abstract:

We focus on the issue of performance evaluation in supply chains, in particular the concept of "Agility". All parameters of agility can be qualitatively judged in linguistic terms. We specifically target the problem of quantification of agility by using fuzzy logic and to develop an index for the same. In particular, we use triangular fuzzy numbers constructed on a standard scale for representing both the weightage and the performance of a particular parameter. We compute a closed form expression of the association function of the proposed performance index which is useful in defuzzification. The closed form expression is very useful for further mathematical treatment and is claimed to be one of central achievements of our work. We also develop a methodology called the Critical Parameter Identification (CPI) method to identify the most important parameters for enhancement in the overall agility index. We illustrate the computation of the closed form expression of the association function, identification of the critical parameter and defuzzification using the Euclidean distance method for a simple 2-parameter example and then generalize the same for N parameters.

Key Words: fuzzy logic, agility, supply chain, Quantification of agility.

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Introduction

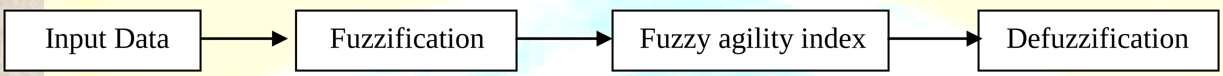
Rapid industrial growth, accelerating technological changes and globalization policies have ushered in intense competition in manufacturing world leading to a shift in focus from individual firms to their respective supply chains. A supply chain is one that forges legally separate but operationally dependent business entities [1], via a feed forward flow of material and feedback flow of information. It comprises a worldwide network of suppliers, factories, warehouses, distribution centers through which raw materials are acquired, transformed and delivered to the end user [2]. Increased complexity of supply networks and changes in business environment like greater thrust on consumer driven demand, product customization, demand responsiveness, market volatility, cost efficiency, sustainability etc. have prompted the need for an effective management of the supply chains. Performance evaluation is at the core of management of complex supply chains. It is an essential element of effective planning/control and also for decision making. It can provide necessary feedback information to reveal progress, enhance motivation and communication and diagnose problems [3]. The quantification of the efficiency and effectiveness of actions provides the necessary feedback for periodic re-adjustments and constant improvements in Supply Chain Management (SCM).

The process of performance evaluation involves first the identification of relevant parameters (done in accordance with the broad objectives of the different supply chain strategies like lean supply chain, agile supply chain, adaptive supply chain etc.) and then their quantification.

Agility is one of the most common and widely used measures of performance evaluation. Agility is described as the ability of a supply chain to rapidly respond to changes in market and customer demands [4,5] and can be analyzed and quantified in terms of four dimensions [6], namely time (to make changes), range (adapting to foreseen and unforeseen changes), intention (reactive or proactive approach) and focus (internal or external restructuring). All companies, suppliers, manufacturers, distributors, and even customers, may have to be involved in the process of achieving an agile supply chain [7, 8]. Using the methods described below, agility is one of the performance measure that can be quantified.

Fuzzy Numbers and Performance Evaluation:

In general the performance levels of all critical parameters in a supply are indicated subjectively by linguistic terms and are characterized by ambiguity. Hence the central issue faced during evaluation is to deal with the inherent vagueness in such qualitative representation of knowledge. Fuzzy logic provides a useful tool to deal with problems which the attributes and phenomenon which are imprecise and vague [9]. In an attempt to use fuzziness, new fuzzy arithmetic [10] has also been proposed. The general steps involved in a fuzzy approach are [1]



An element of a fuzzy set or a fuzzy number is characterized by the range of values and the membership function (which may be linear (triangular, trapezoidal etc) or non linear depending on the nature of system being studied).Generally triangular fuzzy numbers are widely used as only the set of min value, mid range value and max value is required to define them [11].

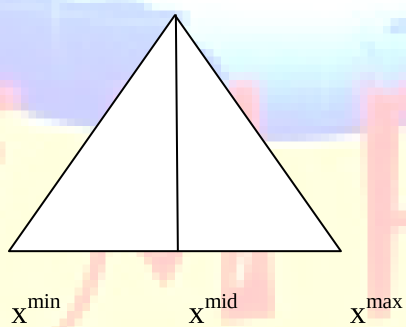


Fig. 1 A Triangular Fuzzy Number

The Fig 1 represents membership function of a triangular fuzzy number (TFN). It means that we are most sure of number taking value x^{mid}. Further the maximum and the minimum values that it can take are x^{min} and x^{max} respectively. This is a good way to represent the vagueness. For example;

The statement "Time to change set up is about 30 mins but it could be as high as 40 mins or as low as 10 mins" can be mathematically quantified as the set up time being a TFN (10, 30, 40).

In the next section we describe how we use the TFN to model performance indices.

Proposed Model:

To quantify performance we propose a performance index that is a weighted sum of various parameters that are considered important for overall performance. The list of these parameters should be obtained from experienced evaluators. We use TFN for the computation of the index as described below. We first describe our model for the weights of various parameters.

1. All weights are assigned a fuzzy value, instead of a crisp value. Fuzzy numbers model the uncertainty in an evaluator's mind. For instance, the weights might be described by linguistic terms like important, essential or insignificant etc. This conversion between linguistic description and triangular fuzzy numbers is briefly described in sec 1.1.

2. Assign a value to each weight, on a standard scale bearing in mind the contribution of the corresponding parameter to the total index independently of other parameters. The value of the weight of this parameter will then represent the relative importance of this parameter with respect to the other parameters.

The following example makes this clear: Let the supply chain be evaluated on the following 5 parameters: N_1: customer satisfaction; N2: delivery time; N3: cost; N4: flexibility; N5: quality. Let the chosen scale be 0 to 10. Let the respective weights be

$\sim W_1$ = (1.5, 2, 2.5); $\sim W_2$ = (2.5, 3, 3.5); $\sim W_3$ = (6.5, 7, 7.5); $\sim W_4$ = (8, 8.25, 8.5); $\sim W_5$ = (9.0, 9.25,9.5)

For i=1,2,…5 $\sim W_i's$ are TFN's.The weights have been defined individually and independently on a scale of 0-10. For example, according to the assessor, parameter 1 has an absolute importance of about 15% to 25% to the final index and parameter an absolute importance of about 25% to 35%. However, for defining this range for parameter 2, the assessor **does not** bear in mind the range of values that have been assigned to parameter 1, all he takes care of is the absolute importance of parameter 2 on a scale of 10. This method inherently also ensures that the relative importance of the parameters is reflected in the assigned values of the weights.

The weights are modeled as TFN's. Similarly to incorporate the uncertainty realized by each parameter we model the realized value ($\sim N_i$) as a TFN. Let $\sim N_i$be the fuzzy value taken by the ith parameter whose weight is given by$\sim W_i$. Then one way of defining the overall index (for a total of n parameters) is:

$$\sim P = \frac{\sum_{i=1}^{n} \sim W_i . \sim N_i}{\sum_{i=1}^{n} \sim W_i} \dots\dots\dots\dots\dots (1)$$

However, if we use only one standard scale for all weights, we can eliminate the normalization parameter $\sum_{i=1}^{n} \sim W_i$ from (1) above. Moreover, when weights are defined on a standard scale it gives a better comparison over different times and scenarios. A standard scale will be easier for the group of assessors to judge these weights. Also in literature the weights are mostly defined on a scale of 0 to 1, which goes well with the framework of a standard scale and hence the problem formulation can be simplified by eliminating the normalization step. Hence we propose the following performance index

$$\sim P = \sum_{i=1}^{n} \sim W_i . \sim N_i \dots\dots\dots\dots\dots (2)$$

Thus the performance index $\sim P$ is a fuzzy number. The rest of the paper deals with computing the membership function of this fuzzy number.

A Two Parameter Model:

We illustrate the method to compute performance with a 2 parameter problem. We assume both weights and performance metrics are triangular fuzzy numbers. This is a very reasonable assumption since assessment in the triangular form is the easiest and might be the only feasible one for assessors. For i = 1, 2, let the performance metric be $\sim N_i$: (n_i^1, n_i^2, n_i^3) and weight be $\sim W_i$: (w_i^1, w_i^2, w_i^3)

Then,

$$\sim P = \{\sim W_1.\sim N_1\} + \{\sim W_2.\sim N_2\} \text{ .... } (3)$$

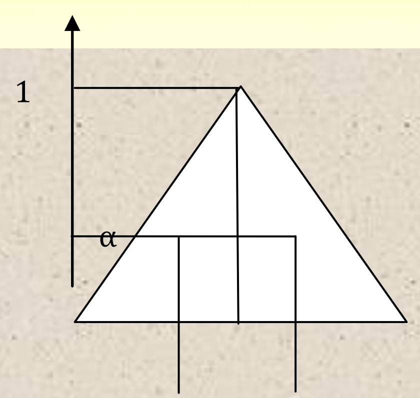
where $\sim W_1$ and $\sim W_2$ are the weights defined on a pre decided scale, and $\sim P$ is the overall performance index. To compute the membership function of $\sim P$ we need some concepts from the fuzzy number theory. We provide a quick review of the same in sec 2.2.

Review of Some Concepts:

Let $\sim Z$ be a fuzzy number with $\mu_Z(z)$ as its membership function.Then the α-cut $\sim Z_\alpha$ of $\sim Z$ is described as follows. The α-cut of a fuzzy number $\sim Z$ is $z \in ({z^l}_\alpha, {z^r}_\alpha)$ such that $\mu_Z(z) \geq \alpha$ as shown in Fig 2.Thus $\sim Z_\alpha$ is an interval on the real line with the lower limit ${z^l}_\alpha$ and the upper limit ${z^r}_\alpha$. We say that the α-cut of $\sim Z$ forms a nested structure with respect to α if

$$[(z)_{\alpha 1}{}^l, (z)_{\alpha 1}{}^r] \subseteq [(z)_{\alpha 2}{}^l, (z)_{\alpha 2}{}^r] \quad \text{for } 0 \leq \alpha 2 \leq \alpha 1 \leq 1 \text{ ....} (4)$$

This is in general true for TFN's as can be observed in Fig 2.



0

z^1 $z^l{}_\alpha$ z^2 $z^r{}_\alpha$ z^3

Fig 2

Let

$$f = z^2 - z^1$$

$$e = z^3 - z^2$$

Then for a given value of α the α-cut is given by

$$z_\alpha{}^l = \alpha f + z^1$$

$$z_\alpha{}^r = z^3 - \alpha e$$

Further if $z^l{}_\alpha$ and $z^r{}_\alpha$ are invertible with respect to α with $L = [z_\alpha{}^l]^{-1}$ *and* $R = [z_\alpha{}^r]^{-1}$ then the membership function of $\sim Z$ i.e. $\mu_Z(z)$ can be constructed as

$$\mu_Z(z) = \begin{cases} L & z^l_{\alpha=0} \le z_\alpha{}^l \le z^l_{\alpha=1} \\ 1 & z^l_{\alpha=1} = z^r_{\alpha=1} \\ R & z^r_{\alpha=1} \le z_\alpha{}^r \le z^r_{\alpha=0} \end{cases} \quad \text{...............} (5)$$

We use these ideas to compute the membership function of $\sim P$ in the next section.

Computation for the two parameter model:

We define

$$f_i = w_i^2 - w_i^1$$

$$e_i = w_i^3 - w_i^2$$

$$d_i = n_i^2 - n_i^1$$

$$g_i = n_i^3 - n_i^2$$

.

Since $\sim W_i$ and $\sim N_i$ are TFN's their α- cuts are nested (equation (4)) and given by$({w_i}_\alpha^l, {w_i}_\alpha^r)$ and $({n_i}_\alpha^l, {n_i}_\alpha^r)$ where

$${w_i}_\alpha^l = \alpha f_i + w_i^1$$

$${w_i}_\alpha^r = w_i^3 - \alpha e_i$$

$${n_i}_\alpha^l = \alpha d_i + n_i^1$$

$${n_i}_\alpha^r = n_i^3 - \alpha g_i$$

Then the α-cut of $\sim P$ is given by $[\, {p_\alpha}^l, {p_\alpha}^r \,]$ where,

$${p_\alpha}^l = \min p = \sum_{i=1}^{n} w_i . n_i$$

s.t

$${w_i}_\alpha^l \leq w_i \leq {w_i}_\alpha^r$$

$${n_i}_\alpha^l \leq n_i \leq {n_i}_\alpha^r \qquad \ldots\ldots\ldots\ldots\ldots (6)$$

$${p_\alpha}^r = \max p = \sum_{i=1}^{n} w_i . n_i$$

s.t

$${w_i}_\alpha^l \leq w_i \leq {w_i}_\alpha^r$$

$${n_i}_\alpha^l \leq n_i \leq {n_i}_\alpha^r \qquad \ldots\ldots\ldots\ldots\ldots (7)$$

Clearly the α-cut of $\sim P$ $(\sim P_\alpha)$ is given by $[{p_\alpha}^l, {p_\alpha}^r]$ where ${p_\alpha}^l$and ${p_\alpha}^r$ are functions of α. Since the α-cut of $\sim P$ is nested, the functions ${p_\alpha}^l$and ${p_\alpha}^r$ are invertible with $L = [{p_\alpha}^l]^{-1}$ and $R = [{p_\alpha}^r]^{-1}$, the association function of $\sim P$ can be constructed as

$$\mu_P(p) = \begin{cases} L & p^l_{\alpha=0} \le p_\alpha{}^l \le p^l_{\alpha=1} \\ 1 & p^l_{\alpha=1} = p^r_{\alpha=1} \\ R & p^r_{\alpha=1} \le p_\alpha{}^r \le p^r_{\alpha=0} \end{cases} \quad \ldots\ldots\ldots\ldots\ldots (8)$$

Clearly,

$$p_\alpha{}^l = w_1{}^l.n_1{}^l + w_2{}^l.n_2{}^l$$

$$p_\alpha{}^r = w_1{}^r.n_1{}^r + w_2{}^r.n_2{}^r \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Plugging in all the values and solving for $p_\alpha{}^l$, we get that

$$p_\alpha{}^l = \beta\alpha^2 + \gamma\alpha + \theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

where,

$$\beta = f_1.d_1 + f_2.d_2$$

$$\gamma = n_1^1.f_1 + w_1^1.d_1 + n_2^1.f_2 + w_2^1.d_2 \quad \ldots\ldots\ldots\ldots\ldots (11)$$

$$\theta = n_1^1 w_1^1 + n_2^1 w_2^1$$

To find the inverse, we solve for α in terms of $p^l(\alpha)$ to get

$$\alpha = \frac{-\gamma \pm \sqrt{\gamma^2 - 4\beta(\theta - p_\alpha{}^l)}}{2\beta}$$

Since all the constants are greater than 0, and α ≥ 0, we can ignore the solution corresponding to the negative sign to get a unique solution *L* as a function of the various values p that can be taken by ~*P*:

$$L = \frac{-\gamma + \sqrt{\gamma^2 - 4\beta(\theta - p_\alpha{}^l)}}{2\beta} \ldots\ldots\ldots\ldots\ldots (12)$$

It is easily seen that $\theta - p_\alpha{}^l$= -$\beta\alpha^2$-$\gamma\alpha$ and hence the entire term under the square root becomes $\gamma^2 + 4\beta^2\alpha^2 + 4\beta\gamma\alpha$, which is always positive (since all β, γ, α are positive) and therefore real roots exist. Similarly, we have

$$p_\alpha{}^r = \alpha^2\beta' - \gamma'\alpha + \theta' \dots\dots\dots\dots\dots (13)$$

, and solving for α, we have

$$\alpha = \frac{\gamma' \pm \sqrt{\gamma'^2 - 4\beta'(\theta' - p_\alpha{}^r)}}{2\beta'}$$

where,

$$\beta' = e_1 g_1 + e_2 g_2$$

$$\gamma' = e_1.n_1^3 + g_1.w_1^3 + e_2.n_2^3 + g_2.w_2^3 \dots\dots\dots\dots\dots\dots..(14)$$

$$\theta' = w_1^3 n_1^3 + w_2^3 n_2^3$$

On simplification, the quantity under the root becomes $\gamma'^2 + 4\beta'^2\alpha'^2 - 4\beta'\gamma'\alpha = (\gamma' - 2\alpha\beta')^2$. Basic algebraic manipulation implies a unique root R of α (the one corresponding to negative sign in the expression of α above) as a function of the various values $p_\alpha{}^r$ that can be taken by ~P:

$$R = \frac{\gamma' - \sqrt{\gamma'^2 - 4\beta'(\theta' - p_\alpha{}^r)}}{2\beta'}$$

It is easily seen that $p^r_{\alpha=1} = p^l_{\alpha=1}$ from the equations above and therefore, using (8), we have a closed form expression for the association function of ~P.

For a 2-parameter model, the next theorem provides a way to compute the membership function of ~P in the general case for n parameter model.

Theorem 2.1(N-Parameter Model):

For n>0 let the final performance measure be defined as $\sim P = \sum_{i=1}^{n} \sim N_i . \sim W_i$*.Then*

$$\mu_P(p) = \begin{cases} L = \dfrac{-\gamma + \sqrt{\gamma^2 - 4\beta(\theta - p_\alpha{}^l)}}{2\beta} & \sum_{i=1}^n w_i^1 n_i^1 \le p_\alpha{}^l \le \sum_{i=1}^n w_i^2 n_i^2 \\ 1 & \sum_{i=1}^n w_i^2 n_i^2 = \sum_{i=1}^n w_i^2 n_i^2 \dots\dots\dots\dots\dots\dots (15) \\ R = \dfrac{\gamma' - \sqrt{\gamma'^2 - 4\beta'(\theta' - p_\alpha{}^r)}}{2\beta'} & \sum_{i=1}^n w_i^2 n_i^2 \le p_\alpha{}^r \le \sum_{i=1}^n w_i^3 n_i^3 \end{cases}$$

where

$$p^l(\alpha) = \beta\alpha^2 + \gamma\alpha + \theta$$

$$p^r(\alpha) = \alpha^2\beta' - \gamma'\alpha + \theta'$$

$$\beta = \sum_{i=1}^{n} f_i . d_i \; ; \; \beta' = \sum_{i=1}^{n} e_i . g_i$$

$$\gamma = \sum_{i=1}^{n}(n_i^1 f_i + w_i^1 d_1) \; ; \gamma' = \sum_{i=1}^{n}(n_i^3 e_i + w_i^3 g_i)$$

$$\theta = \sum_{i=1}^{n} w_i^1 n_i^1 \; ; \theta' = \sum_{i=1}^{n} w_i^3 n_i^3$$

$$L = [p_\alpha{}^l]^{-1} \text{ and } R = [p_\alpha{}^r]^{-1}$$

Proof:

The proof follows from the elementary induction on the two parameter model.

De-Fuzzification:

These equations give the exact expression of the association function of the final index as an explicit function of the original parameters of the fuzzy numbers. The final fuzzy number P representing the overall agility index of the supply chain can be then de-fuzzified using any standard methods like Euclidean distance method, successive approximation method as recommended by Lin et al (2004).We will illustrate the Euclidean distance method on a numerical example in sec. 3.1.

Critical Parameter Identification Method:

In this section we describe a method to choose the most important parameter i.e. the critical parameter, an improvement in which will lead to the greatest enhancement in the overall performance index. We define a framework for index maximization as follows. Two parameters contribute to increase in value of the index:

1. Increase in the value for which the association function is 1 i.e. the mid value
2. Decrease in the “range” of the fuzzy number

Incorporating both the above parameters, we define a Critical Parameter Identifier (CPI) function:

$$Y(\sim W_i, \sim N_i) = \mu\{p^r_{\alpha=0} - p^l_{\alpha=0}\} - (1-\mu)\{p^r_{\alpha=1}\} \ldots\ldots\ldots\ldots\ldots (16)$$

where μ Є (0, 1) is the relative importance of reducing the range. Thus, the performance index improves if the CPI decreases. In order to identify the critical parameter, we define $N_i + \delta = (n_i^1 + \delta, n_i^2 + \delta, n_i^3 + \delta)$ and the derivative $\frac{\partial Y}{\partial N_1}$ of the CPI w.r.t. parameter *i* as

$$\frac{\partial Y}{\partial N_1} = \lim_{\delta \to 0} \frac{Y(\sim W_i, \sim N_i + \delta) - Y(\sim X_i, \sim N_i)}{\delta}$$

We find this derivative for each of the parameters N_i and conclude that the critical parameter is the one for which the derivative is the least. The parameter μ is an input that should be supplied by the assessor and can be taken to be ½ in the absence of any other input.

Numerical Example:

We will now illustrate the procedure clear by taking a 2 parameter model example:

Member Function Calculation:

Let: $W_1 = (1, 2, 3)$, $W_2 = (7, 8, 9)$; $N_1 = (4, 5, 6)$, $N_2 = (1, 2, 3)$

Evaluating all the constants:

$$\beta = 2, \gamma = 13, \theta = 11, \beta' = 2, \gamma' = 21, \theta' = 45$$

From (9), we have

$$p_\alpha{}^l = 2\alpha^2 + 13\alpha + 11$$

$$p_\alpha{}^r = 2\alpha^2 - 21\alpha + 45$$

Using (15) we have

$$\mu_P(p) = \begin{cases} L = \dfrac{-13+\sqrt{169-8(11-p_\alpha{}^l)}}{4} & 11 \le p_\alpha{}^l \le 26 \\ 1 & p_\alpha{}^l = p_\alpha{}^r = 26 \\ R = \dfrac{21-\sqrt{441-8(45-p_\alpha{}^r)}}{4} & 26 \le p_\alpha{}^r \le 45 \end{cases}$$

Defuzzification by Euclidean Distance Method:

The final performance index obtained is a fuzzy number and has to be de-fuzzified to be converted into a linguistic expression. For this we need a set of fuzzy numbers representing the linguistic labels which should be obtained from the assessors. For many purposes, it is sufficient to have the following labels:

DA: Definitely Agile, EA: Extremely Agile, VA: Very Agile, HA: Highly Agile, A: Agile, F: Fairly, SA: Slightly Agile, LA: Low Agile, S: Slowly Agile. These are represented in Fig 3.

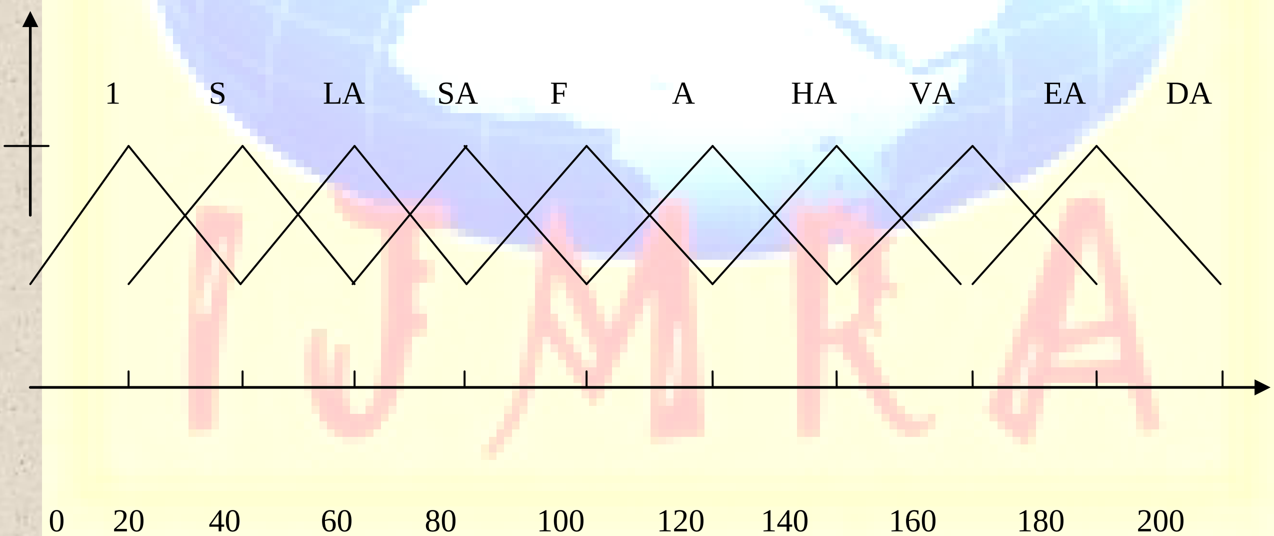


Fig 3

Looking at the end points of the final performance index ~P it can be concluded that the linguistic label can be slowly agile, low agile or slightly agile. Taking the interval for the calculation as 10 and calculating the Euclidean distances, we get

D ($\sim P$, S) = .7203

D ($\sim P$, LA) = 1.0215

D ($\sim P$, SA) = 1.5973

Since the Euclidean distance is minimum for S, we conclude that the supply chain is slowly agile.

CPI Method:

Calculating the function Y (from 16): $Y(W_i, N_i) = \mu\{45 - 11\} - (1-\mu)\{2 + 13 + 11\}$

$$= 34\mu - (1-\mu)26$$

Now, consider an increment in parameter 1. $N_1: (7, 8, 9) \rightarrow (7+\delta, 8+\delta, 9+\delta)$. Re-evaluating all constants:

$$\beta = 2, \gamma = 13 + \delta, \theta = 11 + \delta, \beta' = 2, \gamma' = 21 + \delta, \theta' = 45 + 3\delta$$

For these new constants, the improvement function will be

$$Y_1 = Y(W_1 + \delta, \ldots) = \mu\{45 + 3\delta - (11 + \delta)\} - (1-\mu)\{2 + 13 + \delta + 11 + \delta\}$$
$$= \mu(34 + 2\delta) - (1-\mu)\{26 + 2\delta\}$$

Differentiating w.r.t. the first parameter,

$$\frac{\partial Y}{\partial N_1} = \frac{Y_1 - Y}{\delta} = \frac{[\mu\{34 + 2\delta\} - (1-\mu)\{26 + 2\delta\}] - [34\mu - (1-\mu)26]}{\delta} = 4\mu - 2$$

Similarly, for a small increment in parameter 2. $N_2: (1,2,3) \rightarrow (1+\delta, 2+\delta, 3+\delta)$

$$\beta = 2, \gamma = 13 + \delta, \theta = 11 + 7\delta, \beta' = 2, \gamma' = 21 + \delta, \theta' = 45 + 9\delta$$

Calculating the improvement function

$$Y_2 = Y(W_2 + \delta, \dots) = \mu\{45 + 9\delta - (11 + 7\delta)\} - (1 - \mu)\{2 + 13 + \delta + 11 + 7\delta\}$$

$$= \mu(34 + 2\delta) - (1 - \mu)\{26 + 8\delta\}$$

Again, differentiating this w.r.t. the second parameter gives

$$\frac{\partial Y}{\partial N_2} = \frac{Y_2 - Y}{\delta} = \frac{[\mu\{34 + 2\delta\} - (1 - \mu)\{26 + 8\delta\}] - [34\mu - (1 - \mu)26]}{\delta} = 10\mu - 8$$

Comparing the derivatives for both parameters, we see that $\frac{\partial Y}{\partial N_2} \leq \frac{\partial Y}{\partial N_1}$ for all values of μ.

Hence, we can conclude that the supply chain performance will improve the most by concentrating efforts on parameter 2.In general, for any value of μ, the company must focus on the parameter that comes out to be most negative. If in any case all the values of the derivatives are positive, then we can conclude that for the chosen value of μ the performance level cannot be improved further.

Conclusions and Further Extension:

The complexity of the proposed model (2) increases manifold by incorporating the normalizing step (refer to the appendix for formulation (eq. 17) and further explanation). Hence by justifiably eliminating that step we are able to develop a simplified framework with a closed form expression for the final fuzzy index. We are also able to identify the critical parameter for performance enhancement using the CPI function. Further extensions can be made by using non triangular fuzzy numbers. Also the proposed method can be applied to a case study.

Appendix:

Computational Advantage of doing away with the normalization parameter:

Including the normalizing step, the performance index is given by:

$$\sim P = \frac{\sum_{i=1}^{n} \sim W_i . \sim N_i}{\sum_{i=1}^{n} \sim W_i} = \sum ((\sim W_i / \sum \sim W_i) * \sim N_i) \quad \text{..............} (17)$$

The membership function of ~*P* is found using the fractional programming approach described in *Kao and Liu, [12]*. For $i = 1,2,\dots,n$, let $\sim W_i$, $\sim N_i$ be fuzzy numbers defined as:

$$\sim W_i = \{(w_i, \mu_{W_i}(w_i)) \ \forall \ w_i \in W_i\}$$

$$\sim N_i = \{(n_i, \mu_{N_i}(n_i)) \ \forall \ n_i \in N_i\}$$

Then the overall index $\sim P = \frac{\sum_i^n \sim W_i . \sim N_i}{\sum_i^n \sim W_i}$ and its membership function is given by

$$\mu_P(p) = \sup_{n_i, w_i} [\min \{\mu_{W_i}(w_i), \mu_{N_i}(n_i) \mid p = \frac{\sum_{i=1}^{n} w_i . n_i}{\sum_{i=1}^{n} w_i}\}] \quad \text{.............} (18)$$

This can be formulated as the following non linear programming problem (NLP):

$$\mu_P(p) = \max z$$

$$s.t$$

$$z \le \mu_{W_i}(w_i),$$

$$z \le \mu_{N_i}(n_i)$$

$$p = \frac{\sum_{i=1}^{n} w_i . n_i}{\sum_{i=1}^{n} w_i}$$

$$w_i \in W_i$$

$$n_i \in N_i, \qquad \text{for } i = 1,2,\dots,n$$

Since the model is non-linear and the association functions non differentiable, it is very difficult to solve the above NLP for the association function $\mu_P(p)$.

A general way to solve for $\mu_P(p)$ is the following: Define α-cuts $(w_i)_\alpha$ and $(n_i)_\alpha$ respectively on $\sim W_i$ and $\sim N_i$ and solve for the α-cut $\sim P$ in terms of $(w_i)_\alpha$ and $(n_i)_\alpha$. If all the α-cuts form a nested structure with respect to α i.e. $[(w_i)_{\alpha 1}^{l}, (w_i)_{\alpha 1}^{r}] \subseteq [(w_i)_{\alpha 2}^{l}, (w_i)_{\alpha 2}^{r}]$ and $[(n_i)_{\alpha 1}^{l}, (n_i)_{\alpha 1}^{r}] \subseteq [(n_i)_{\alpha 2}^{l}, (n_i)_{\alpha 2}^{r}]$ for $0 \le \alpha 2 \le \alpha 1 \le 1$, then $\sim P_\alpha$ is an interval on the real number line given by $[p_\alpha{}^l, p_\alpha{}^r]$ where both $p_\alpha{}^l$ and $p_\alpha{}^r$ are functions of α. Further, if the functions are invertible with $L = [p_\alpha{}^l]^{-1}$ and $R = [p_\alpha{}^r]^{-1}$, the membership function of $\sim P$ can be constructed as given in (8).Kao and Liu (2001) suggest the following method to find $p_\alpha{}^l$ and $p_\alpha{}^r$: The α-cuts on $\sim W_i$ and $\sim N_i$ are given by

$$(w_i)_\alpha = \{w_i \in \sim W_i \text{ s.t. } \mu_{W_i}(w_i) \ge \alpha\} = [\, {w_i}_\alpha^l, {w_i}_\alpha^r]$$

$$(n_i)_\alpha = \{n_i \in \sim N_i \text{ s.t. } \mu_{N_i}(n_i) \ge \alpha\} = [\, {n_i}_\alpha^l, {n_i}_\alpha^r]$$

Then,

$$p_\alpha{}^l = \min p = \frac{\sum_{i=1}^{n} w_i.n_i}{\sum_{i=1}^{n} w_i}$$

s.t

$${w_i}_\alpha^l \le w_i \le {w_i}_\alpha^r$$

$${n_i}_\alpha^l \le n_i \le {n_i}_\alpha^r$$

and,

$$p_\alpha{}^r = \max p = \frac{\sum_{i=1}^{n} w_i.n_i}{\sum_{i=1}^{n} w_i}$$

s.t

$${w_i}_\alpha^l \le w_i \le {w_i}_\alpha^r$$

$${n_i}_\alpha^l \le n_i \le {n_i}_\alpha^r$$

To solve for ${p_\alpha}^l$ and ${p_\alpha}^r$ we

1. Substitute $n_i = {n_i}_\alpha^l$ in the objective function of min formulation and $n_i = {n_i}_\alpha^r$ in the max formulation, call the resultant expressions p^l and p^r respectively
2. After this substitution, find the gradient of p^l and p^r w.r.t. a particular parameter say w_k and check the sign, i.e.

$$\partial p^l / \partial w_k = \frac{\sum_{i=1}^{n} w_i ({n_k}_\alpha^l - {n_i}_\alpha^l)}{\left[\sum_{i=1}^{n} w_i\right]^2}$$

$$\partial p^r / \partial w_k = \frac{\sum_{i=1}^{n} w_i ({n_k}_\alpha^r - {n_i}_\alpha^r)}{\left[\sum_{i=1}^{n} w_i\right]^2}$$

If the sign of the gradient is positive then substitute $w_k = {w_i}_\alpha^l$ and $w_k = {w_i}_\alpha^r$ for the min or max formulation respectively. The sign of the gradients may change depending on the value of α. This has to be now repeated for all other parameters. These expressions vary across different ranges of α. This procedure becomes complex as the number of parameters increase.

References:

- Ching-Torng Lin, Hero Chiu, Po-Young Chu, Agility index in supply chain, Int. J. Production Economics 100 (2006) 285-299
- Lin, C., T.H. Chiu and Y.H. Tseng, 2006. Agility evaluation using fuzzy logic. Int. J. Prod. Econ., 1: 353-368. DOI: 10.1016/j.ijpe.2004.11.013
- Felix T.S. Chan, H.J. Qi, H.K. Chan, Henry C.W. Lau, Ralph W.L. Ip, A conceptual model of performance measurement for supply chains.
- Yusuf Y.Y., Gunasekaran, Adeleye, Sivayoganathan, 2004. Agile supply chain capabilities: determinants of competitive objectives. European Journal of Operational Research 159, 379-392.
- Yusuf, Sarhadi, Gunasekaran, 1999. Agile Manufacturing: the drivers, concepts and attributes. International Journal of Production Economics 62, 34-43.
- William Golden, Philip Powell, Towards a definition of flexibility: in search of the Holy Grail? Omega 28 (2000) 373-384.

- Christopher, 2000. The agile supply chain: competing in volatile markets. Industrial marketing Management 29 (1), 37-44.
- Christopher, Towill, 2001. An integrated model for design of agile supply chains. International Journal of Physical distribution and Logistics 31 (4), 234-246.
- Zadeh, L.A., Fuzzy sets. Information and Control 8 (3), 338-353.
- Ranjan Alex, Fuzzy point estimation and its application on fuzzy supply chain analysis, Fuzzy Sets and Systems 158 (2007) 1571 – 1587
- George J. Klir/ Bo Yuan, Fuzzy Sets and Fuzzy Logic Theory And Applications.
- Chiang Kao, Shiang-Tai Liu, 2001. Fractional programming approach to fuzzy weighted average. Fuzzy sets and systems 120, 435-444.

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